On the Mercury Green Line λ = 5461 *Resolved by Glass and Quartz Lummer Plates and on its Zeeman Constituents.*

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[Plates 3 and 4.]

Numerous investigators have studied the structure of some of the finer lines in the mercury arc, and other spectra, with ruled gratings, échelon spectroscopes, and Lummer plates; but up to the present when Lummer plates and échelons were used these were invariably made of glass. A short time ago a Lummer plate of crystal quartz was made for the Physical Laboratory at Toronto by the Adam Hilger Company, for the investigation of the structure of some of the finer lines in ultra-violet spectra; but before proceeding to use it for this purpose some measurements were made with it, and with two Lummer plates made of glass, on the wave-lengths of the satellites of the mercury green line $\lambda = 5461$ Å.U. At the same time a precise method of measuring up the photographic fringe patterns produced by Lummer plates has been developed in which the geometrically central line of the double fringe pattern is taken as the line of reference in the measurements. This exact formula permits of the use of the fringes that emerge from the plate at nearly grazing direction, for which the dispersion is the greatest. The following paper contains an account of the investigation and includes the values of the wave-lengths of the satellites of the green line as deduced by the application of the method to which reference has just been made. The quartz Lummer plate, it may be added, was cut with the axis of the crystal parallel to the plane refracting surfaces and perpendicular to the length of the plate.

The source of the light used in the investigation was one or other of a number of ordinary Cooper-Hewitt mercury-arc lamps. The light was passed through the collimator tube of a spectroscope and then into the plate in the usual manner.

In studying the structure of the line the Lummer plates were always placed with their plane parallel faces vertical so that the light which issued from both plane faces was available for the production of the fringe pattern. The light as it issued from the plate was allowed to fall upon the lens of a camera placed behind the Lummer plate, with the central plane of the lens perpendicular to the Lummer plate and its principal axis directed along the central plane of the latter. By this arrangement the beams of parallel rays which issued at grazing emergence from both sides of the plate were brought to a focus in the line in which the central plane of the Lummer plate cut the focal plane of the lens, while beams which issued in directions inclined at an angle to the plate were brought to a focus in the focal plane either to the right or to the left of the central line according as they issued from one side or the other of the Lummer plate.

Photographs taken in this way of the fringe pattern produced by the Lummer plates with the light of the mercury green line include a set of wide main line fringes, and a number of sets of narrow fringes corresponding to the different satellites which accompany the main line.

It is easy to show from the theory of Lummer plates that the fringes constituting any one of those sets are spaced out according to a very simple law. If, for example, we consider the main line set and take d to be the thickness of the Lummer plate, μ its refractive index for the wave-length λ, and b the focal length of the camera lens, and denote by $2\alpha_1, 2\alpha_2, \ldots, 2\alpha_n$, the distances between the centres of the main line fringes taken in pairs, one being on either side of the central line of the photograph, then it can be shown that

$$\lambda = \frac{d}{b^2} \cdot \frac{\alpha_n{}^2 - \alpha_{n-1}{}^2}{\sqrt{(\mu^2-1)}}, \tag{1}$$

so that the quantity $\alpha_n{}^2 - \alpha_{n-1}{}^2$ should to a very close degree have the same value over the whole fringe pattern, *i.e.* denoting $\alpha_{n+m}{}^2 - \alpha_n{}^2$ by Ω_m, and $\alpha_n{}^2 - \alpha_{n-1}{}^2$ by Ω_1, we should have for the whole pattern

$$\Omega_m/m = \Omega_1 = \text{a constant.} \tag{2}$$

For let the two systems of emergent parallel rays which go to form bright fringes of the nth and $(n+m)$th orders issue from the plate at the angles ψ and ψ' respectively. Then, denoting the path differences corresponding to these two fringes by γ and $\gamma + m\lambda$, we have

$$\gamma = 2d\sqrt{(\mu^2 - \cos^2\psi)}, \quad \text{and} \quad \gamma + m\lambda = 2d\sqrt{(\mu^2 - \cos^2\psi')}.$$

From these two equations we have

$$m\lambda = 2d\,\frac{\cos^2\psi - \cos^2\psi'}{\sqrt{(\mu^2 - \cos^2\psi')} + \sqrt{(\mu^2 - \cos^2\psi)}};$$

or, if the angles ψ and ψ' are small,

$$m\lambda = \frac{d}{b^2} \cdot \frac{\alpha_{n+m}{}^2 - \alpha_n{}^2}{\sqrt{(\mu^2-1)}}.$$

Numerous tests were made of the law given by equation (2) by measuring up a number of photographs of the fringe patterns obtained with the different Lummer plates, and it was found to hold invariably with very great exactness. One particular case in which it was possible to measure the distances between the bands corresponding to 11 different orders on the plate may be cited. The values of $\alpha_1, \alpha_2, \ldots, \alpha_n$, and $\alpha_1^2, \alpha_2^2, \ldots \alpha_n^2$ for this plate are given in Table I, from which it will be seen that the latter are in arithmetical progression, so that the values of Ω_1 deduced from $\Omega_1 = \Omega_m/m$ are in close agreement, and the most probable value of Ω_1 can be deduced by well-known methods. The uniformity exhibited in the values of Ω_1 shows decisively that the law enunciated above represents exceedingly closely the distribution of the fringes in the Lummer plate pattern.

Table I.—Determination of Ω_1 from 11 Orders of Main Line Fringes obtained with a glass Lummer plate, by means of Equation (2).

	Difference.		Difference.
$\alpha_1 = 0{\cdot}5697$		$\alpha_1^2 = 0{\cdot}3246$	
$\alpha_2 = 0{\cdot}8428$	0·2731	$\alpha_2^2 = 0{\cdot}7103$	0·3857
$\alpha_3 = 1{\cdot}0478$	0·2050	$\alpha_3^2 = 1{\cdot}0979$	0·3876
$\alpha_4 = 1{\cdot}2194$	0·1716	$\alpha_4^2 = 1{\cdot}4870$	0·3891
$\alpha_5 = 1{\cdot}3677$	0·1483	$\alpha_5^2 = 1{\cdot}8706$	0·3836
$\alpha_6 = 1{\cdot}5022$	0·1345	$\alpha_6^2 = 2{\cdot}2566$	0·3860
$\alpha_7 = 1{\cdot}6264$	0·1242	$\alpha_7^2 = 2{\cdot}6452$	0·3886
$\alpha_8 = 1{\cdot}7407$	0·1143	$\alpha_8^2 = 3{\cdot}0300$	0·3848
$\alpha_9 = 1{\cdot}8488$	0·1081	$\alpha_9^2 = 3{\cdot}4181$	0·3881
$\alpha_{10} = 1{\cdot}9499$	0·1011	$\alpha_{10}^2 = 3{\cdot}8021$	0·3840
$\alpha_{11} = 2{\cdot}0460$	0·0961	$\alpha_{11}^2 = 4{\cdot}1862$	0·3839

In this connection it may be pointed out that for lines whose wavelengths are so close together as those of the main component and those of the satellites of the mercury green line the value of Ω_1 should come out about the same, when deduced either from measurements on the fringes of the main line system or from those made on the fringes of a satellite system. That this is true may be seen from the values of Ω_1 given in Table II, which were deduced from measurements similar to those described above made upon the same plate on the fringe system of positive satellite 0·093, of positive satellite 0·138, of negative satellite 0·07, as well as on the system of fringes due to the main component.

Table II.

Fringe pattern of	Deduced value of Ω_1.
Satellite 0·138	0·3860
Satellite 0·093	0·3862
Main component	0·3863
Satellite −0·07	0·3856

The formula referred to above as the one used by us in determining the differences between the wave-lengths of the different satellites, and the wave-length of the main constituent of the green line, is expressible in the form

$$\Delta\lambda = \frac{\delta(\alpha^2)}{\Omega_1}\Delta\lambda_m, \tag{3}$$

where Ω_1 has the significance given to it above, and is equal to the difference between the squares of the distances of two consecutive main line fringes from the central line of the fringe pattern, *i.e.*, $\alpha_{m_1}{}^2 - \alpha_m{}^2$ and $\delta(\alpha^2)$ denotes either the expression $\alpha_{m_1}{}^2 - \alpha_s{}^2$ or $\alpha_s{}^2 - \alpha_m{}^2$, where α_m is the distance of a main line fringe from the central line of the pattern, α_{m_1} that of the main line fringe of the next higher order, and α_s is the distance from the central line of a satellite fringe of the same order as α_m. $\Delta\lambda_m$ represents that change in the wave-length of the light which would cause the fringe of the mth order of the system of fringes due to light of wave-length $\lambda + \Delta\lambda_m$ to coincide with the fringe of the $(m+1)$th order of the system formed by the light of wave-length λ.

The value of $\Delta\lambda_m$, as v. Baeyer* has shown, is given by

$$\Delta\lambda_m = \frac{\lambda^2 q}{q^2\lambda - 4d^2\mu\, d\mu/d\lambda}, \tag{4}$$

where

$$q = \frac{2d}{\lambda}\sqrt{(\mu^2-1)} \tag{5}$$

and is calculable for any wave-length λ when the thickness of the Lummer plate, and its refractive index as well as its dispersion for the wave-length λ, are known.

The validity of the equation $\Delta\lambda = \frac{\delta(\alpha^2)}{\Omega_1}\Delta\lambda_m$ can be seen by throwing it into the form

$$\alpha_s{}^2 = \frac{\Omega_1}{\Delta\lambda_m}\Delta\lambda + \alpha_m{}^2, \tag{6}$$

where α_s and $\Delta\lambda$ are taken to be the variables.

In this form the relation shows that in the interference point-pattern formed by crossing a Lummer plate with an échelon grating, the points corresponding to the different satellites of any selected order all lie on one parabola. The beautiful diagrams accompanying the paper by Nagaoka and Takamine† amply illustrate this point.

In using the formula $\Delta\lambda = \frac{\delta(\alpha^2)}{\Omega_1}\Delta\lambda_m$ for determining the wave-length differences $\Delta\lambda$ for the different satellites of a main line the quantities to be determined by measurement are Ω_1, viz., $\alpha_{m_1}{}^2 - \alpha_m{}^2$, and $\delta(\alpha^2)$, *i.e.*, $\alpha_s{}^2 - \alpha_m{}^2$. As we have pointed out above, Ω_1 is found by measuring the distance between pairs of corresponding main line or satellite fringes, one member of each pair being on either side of the double pattern. Similar measurements made on pairs of any selected satellite's fringes will give the values of $\alpha_s{}^2$ for the different orders for this satellite. The quantity $\alpha_s{}^2 - \alpha_m{}^2$ can then be found. Since this quantity, as equation (3) shows, should have the same value for all orders, the mean of its values taken over all the orders in the fringe pattern should give a very accurate result.

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* v. Baeyer, 'Verh. d. Deutsch. Phys. Ges.,' p. 734, No. 18/20, 1908.

† Nagaoka and Takamine, 'Phys. Soc. Lond. Proc.,' vol. 25, pt. 1, Dec. 15, 1912.

In the present investigation the formula $\Delta\lambda = [\delta(\alpha^2)/\Omega_1]\Delta\lambda_m$ was used in deducing $\Delta\lambda$ for the different satellites of the mercury green line from measurements on eight different photographs taken with the quartz plate, and all the results are recorded in Table III, wave-lengths being given in Ångström units. The values obtained from the different photographs are all in good agreement. It will be seen that the fringes due to the negative satellite No. 5 found by one of us* with an échelon grating and by a number

Table III.

Data for quartz Lummer plates.

$d = 0{\cdot}47$ cm.

$\lambda = 5461 \times 10^{-8}$ cm.

$\mu = 1{\cdot}55537$

$d\mu/d\lambda = -528^\circ$

$\Delta\lambda_m = 0{\cdot}2582$ Å.U.

Refractive indices.	
λ.	μ.
7065·59	1·54949
6563·04	1·55095
5893·17	1·55337
5607·1	1·55462
5270·11	1·55639
4861·49	1·55899

Plate.	Negative satellites.			Positive satellites.		
	No. 5.	No. 4.	No. 3.	No. 1.	No. 2.	No. 3.
No. 1 …………	…	−0·108	−0·075	0·079	0·1195	0·206
,, 2 …………	…	−0·108	−0·075	0·0795	0·1203	0·208
,, 3 …………	…	−0·107	−0·074	0·0795	0·1204	0·207
,, 4 …………	…	−0·110	−0·078	0·0794	0·1184	0·204
,, 5 …………	…	−0·108	−0·073	0·0817	0·1185	0·207
,, 6 …………	…	−0·107	−0·077	0·0781	0·1216	0·205
,, 7 …………	…	−0·107	−0·075	0·0789	0·1198	0·208
,, 8 …………	…	−0·104	−0·074	0·0788	0·1230	0·207
Mean ………	…	−0·107	−0·075	0·0794	0·1202	0·206

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* McLennan, 'Roy. Soc. Proc.,' A, vol. 87, p. 269 (1912).

of other experimenters at about −0·243 Å.U. did not appear on the photographs taken with the quartz plate. This was because the fringe for this satellite of any order always coincided in the pattern with the main line fringe of the next lower order.

A similar uniformity marked the results obtained from measurements made on a great many photographs taken with the glass Lummer plates. Illustrations of these are given in Table IV, which contains the values of $\Delta\lambda$ in Å.U. for the different satellites found from two photographs taken with one of the glass plates, and from one taken with the other. No values are given for the $\Delta\lambda$ of positive satellite No. 3, situated at about 0·210 Å.U., as the fringes for it did not appear on any of the photographs, owing very probably to their overlapping or lying very close to those of negative satellite No. 3.

Table IV.

Data for glass Lummer plates.

$d = 0{\cdot}448$ cm.

$\lambda = 5461 \times 10^{-8}$ cm.

$\mu = 1{\cdot}5121$

$d\mu/d\lambda = -530$

$\Delta\lambda_m = 0{\cdot}2984$ Å.U.

Refractive indices.	
λ.	μ.
6563·045	1·50746
5896·155	1·50990
5890·186	
4861·49	1·51560
4308·08	1·52025

	Negative satellites.			Positive satellites.		
	No. 5.	No. 4.	No. 3.	No. 1.	No. 2.	No. 3.
Lummer Plate A—						
Pattern No. 1	−0·245	−0·108	−0·069	0·092	0·137	
„ „ 2	−0·246	−0·109	−0·071	0·092	0·137	
Lummer Plate B—						
Pattern No. 1	−0·244	−0·108	−0·069	0·094	0·139	
Mean	−0·245	−0·108	−0·070	0·093	0·138	

The mean values of $\Delta\lambda$ for the different satellites of the mercury green line deduced from those in Tables III and IV are collected in Table V, and along with them are also given a number of determinations of the $\Delta\lambda$ for the satellites of the line found by other investigators with different types of resolving apparatus. From this table it will be seen that Nagaoka and Takamine were able to resolve the main line into a doublet with −0·026 as the position of the constituent of shorter wave-length. As the table shows, the main line was not resolved into a doublet either by the quartz plate or by the glass plates used by us. Nagaoka and Takamine also give a faint satellite at about −0·054. This satellite, it will be noted, was not distinguishable on any of our photographs, either in the work with the Lummer plates or in that done by one of us with an échelon. Moreover, it does not appear among the list of satellites for the line given by Gale and Lemon in the report of their investigation of the structure of the line with a Michelson grating. From the character of Nagaoka and Takamine's work, however, it would appear that this satellite has a real existence.

Table V.—Values of $\Delta\lambda$ for Satellites of Mercury Green Line $\lambda = 5461$ Å.U.

Resolution by	Negative satellites.					Main line.	Positive satellites.		
	No. 5.	No. 4.	No. 3.	No. 2.	No. 1.		No. 1.	No. 2.	No. 3.
McLennan and McLeod—									
Glass Lummer plates	−0·245	−0·108	−0·070	...	...	0	0·093	0·138	
Quartz Lummer plate	...	−0·107	−0·075	...	...	0	0·079	0·120	0·206
McLennan*—									
Echelon	−0·243	−0·104	−0·070	...	...	0	0·090	0·135	0·210
Gale and Lemon†—									
Michelson grating	−0·246	−0·107	−0·067	...	...	0	0·070	0·117	0·204
Nagaoka and Takamine‡—									
Echelon	−0·246	−0·113	−0·080	−0·055	−0·026	0	0·072	0·118	0·204
Echelon crossed with Lummer plate	−0·242	−0·108	−0·074	−0·054	−0·026	0	0·078	0·123	0·210

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* McLennan, 'Roy. Soc. Proc.,' A, vol. 87, p. 269 (1912).

† Gale and Lemon, 'Ast. Phys. Journ.,' January, 1910; 'Phys. Zeit.,' vol. 11, p. 209 (1910).

‡ Nagaoka and Takamine, 'Phys. Soc. Proc.,' London, p. 1, December 15, 1912.

The positions of the negative satellites Nos. 3, 4, and 5, as found by the various types of apparatus, it will be seen from the table are all in good agreement. A discrepancy exists, however, in regard to the positions of the positive satellites Nos. 1 and 2. The glass Lummer plates used by the writers, and the échelon resolution obtained by one of us, give these satellites at about 0·092 and 0·137, while the resolutions by Gale and Lemon, by Nagaoka and Takamine, and by the writers with the quartz Lummer plate, give the positions of these two satellites at about 0·079 and 0·120. In order, if possible, to remove this discrepancy a number of photographs of the fringe patterns formed both by the quartz Lummer plate and by the two glass ones were taken with the light from Cooper-Hewitt lamps of diameters ranging from 1 to $2\frac{1}{2}$ cm. run with current intensities of from 4 to 7 ampères, but the positions of these two satellites always came out, when the plates were measured up, in close agreement with the numbers given in Tables III and IV.

In one particular case a photograph of the fringe pattern formed by the quartz Lummer plate was taken simultaneously with the one formed by one of the glass Lummer plates, when the collimators of the spectroscopes carrying those two plates were directed at the same portion of the arc in the Cooper-Hewitt lamp. The values of $\Delta\lambda$ found for positive satellites Nos. 1 and 2 from the pattern formed by the glass plates were respectively 0·093 and 0·137 while the values found from the photograph taken with the quartz Lummer plate were 0·081 and 0·124. This result would go to show that the discrepancy did not arise from any real variation in the structure of the line. It will be seen, moreover, that while the resolution by the glass plate gave the wave-length of positive satellite No. 2 as 0·044 longer than that of positive satellite No. 1, that by the quartz plate gave it as 0·043 longer, which is practically the same. This might seem to indicate that the discrepancy arose from an error in setting the micrometer on the main line fringes in measuring up the plates. Such an error, however, should affect the values of $\Delta\lambda$ for all the satellites in a similar way and should not introduce any error in the determination of the wave-lengths of the positive satellites 1 and 2 relative to those of any of the other satellites. The results in Table V show, however, that while the values of $\Delta\lambda$ for these two satellites relative to each other are the same, for both the glass and quartz plates, their values relative to those of negative satellites Nos. 1 and 2, for example, are considerably different for the two kinds of plates.

It may be pointed out that all the photographs from which the values of $\Delta\lambda$ given in Table III were obtained were taken with light emitted by the plate which belonged to the extraordinary ray system. A few photographs were also taken of fringe patterns formed by the ordinary ray system and these were all found to give values for $\Delta\lambda$ for the different satellites practically the same as those given in Table III. In one particular instance a photograph was taken of the pattern formed by the extraordinary rays and immediately afterwards one formed by the ordinary system. When these two plates were measured up to find $\Delta\lambda$ for positive satellite No. 1 its value came out from the ordinary ray pattern as 0·078 and from the extraordinary ray pattern as 0·080. From these considerations it will be seen that the explanation of the discrepancy is by no means evident. All the measurements were made with extreme care and the calculations were always carefully checked over. As pointed out in Table III, the value of $\Delta\lambda_m$ was calculated to be 0·2582 for the fringe system formed by the extraordinary rays. For the ordinary ray system of fringes its value was 0·2608. As these values differ by only 1 per cent., it is clear that if any error were made by inadvertently assuming a pattern to be formed by the extraordinary rays, for example, when it was really made by the ordinary ones, the value found for $\Delta\lambda$ would still be correct to within one unit in the third decimal place. It is interesting to note that the values of $\Delta\lambda$ for the satellites found by us with the quartz plate are extremely close to those found by Nagaoka and Takamine with a glass plate, when the system of measurement used by them was presumably different from the one used by us.

Relations between the Positions of the Satellites.

In dealing with the origin of satellites Hull* has pointed out that a periodicity exists in the spacing of the satellites of some of the mercury lines, which one is scarcely warranted in considering to be accidental. Nagaoka† in an earlier communication had also referred to this matter and in his later paper in conjunction with Takamine (*loc. cit.*) he has again dealt with the subject. Although it has been shown that the values of $\Delta\lambda$ for some of the satellites of the mercury lines 5461, 4078 and 4047 are in a simple ratio, it does not appear to have been noted that with the negative satellites of the line 5461 the $\Delta\lambda$ for any one satellite can be obtained by simply taking either three-halves or nine-fourths of the $\Delta\lambda$ for the satellite next closer than it to the main line.

This may be seen from the results collected in Table VI.

With the positive satellites of the line 5461 Å.U., the 3 : 2 ratio also applies, as Table VI shows, to the values of $\Delta\lambda$ found for these satellites by the writers with the glass Lummer plates, and by one of us with an échelon grating, but for the values of $\Delta\lambda$ for the positive satellites found by Nagaoka and Takamine and also by the writers with the quartz Lummer plate, it will be seen that while the values of the $\Delta\lambda$ for satellites Nos. 1 and 2 are in the ratio 2 : 3, the values of $\Delta\lambda$ for satellites Nos. 2 and 3 are more nearly in the ratio 3 : 5.

The suggestion of the existence of such a law as that mentioned regulating the distribution of the satellites, brings with it the notion that possibly the main constituent of the green line itself might be shown with suitable conditions and apparatus to consist of a great number of very close lines. Some experiments recently made by Janicki,‡ moreover, afford some grounds for this idea. In these the source of the light was an Arons-Lummer mercury lamp, which was kept very cold and was driven with a direct current of 2·5 ampères. The circuit also had included in it a coil of high self-induction.

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* Hull, 'Astrophys. Journ.,' vol. 32, p. 226 (1910).

† Nagaoka, 'Phys. Zeit.,' vol. 10, p. 609 (1909).

‡ Janicki, 'Ann. der Phys.,' vol. 39, p. 439 (1912).

Table VI.—Satellites of Mercury Line λ = 5461 Å.U.

Satellites.	Calculated.	Observed.	
		Nagaoka and Takamine.	McLennan and McLeod.
	Ratio 3 : 2.		Glass and quartz plates.
Negative No. 5	−0·245	−0·242	−0·245
	−0·163		
„ „ 4	−0·109	−0·108	−0·108
			−0·107
„ „ 3	−0·073	−0·074	−0·070
			−0·075
„ „ 2	−0·048	−0·054	
	−0·036		
„ „ 1	−0·024	−0·026	
			Glass plate.
Positive No. 3	0·210		
„ „ 2	0·140		0·138
„ „ 1	0·093		0·093
	Ratio		Quartz plate.
Positive No. 3	0·210	0·210	0·206
„ „ 2	0·126 5 : 3	0·123	0·120
„ „ 1	0·084 3 : 2	0·078	0·079

When the light from this lamp was examined with a system consisting of a Lummer-Gehrcke plate 0·971 cm. thick and 30 cm. long, crossed with a wedge-shaped plate 0·628 cm. thick possessing a wedge angle of 40″, Janicki found that the main component of the green line consisted of a series of five well-marked, very close lines. Thus the suggestion is open that the green line consists of a series of lines closing together towards the centre, their positions being regulated by some law which in the outer components simplifies approximately to the geometric progression exhibited in Table VI.

It is interesting to note that a similar law having the ratio 3 : 5 as a basis, approximately holds in regard to the distribution of the satellites of the mercury line λ = 3341. This line belongs to the same series as the line λ = 5461, and is next in order to it. But little attention has been given as yet to the structure of this line except by G. Wendt,* who examined it with a Rowland grating, and reports it as consisting of a strong main component accompanied by four satellites of wave-length greater and by four of wave-length shorter than the main component. These are all collected in Table VII, and it will be seen that while the departures from the law are more marked than in the case of the line λ = 5461, there is a fairly good agreement between the observed values of $\Delta\lambda$ and those calculated on the basis of the 3/5 law.

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* Wendt, 'Ann. der Phys.,' Folge IV, No. 37, p. 535 (1913).

Table VII.—Satellites of the Mercury Line $\lambda = 3341$ Å.U

Satellites.	Values of $\Delta\lambda$ calculated on basis of ratio 5 : 3.	Values of $\Delta\lambda$ observed by Wendt.
Negative satellites—		
No. 4	−0·600	−0·592
„ 3	−0·360	−0·357
„ 2	−0·216	−0·198
„ 1	−0·130	−0·101
Positive satellites—		
No. 4	0·500	0·497
„ 3	0·300	0·299
„ 2	0·180	0·181
„ 1	0·108	0·127

The Zeeman Effect.

In the course of the investigation into the structure of the green line $\lambda = 5461$ with the Lummer plates, it was noticed that these plates provided an excellent means of illustrating the Zeeman effect with this line and of determining from it the ratio e/m by a method of coincidence.

In making the observations the Cooper-Hewitt lamp was placed longitudinally in the field of an electromagnet provided with pierced pole-pieces. The mechanical details of the manner in which this was done have already been described in a previous communication by one of us.*

The light emitted by the lamp in a direction at right angles to the field was passed through a collimator and into the Lummer plate in the usual way. On emerging from the Lummer plate, the light was then passed through a Wollaston double-image prism and afterwards allowed to fall upon the lens of the camera. With this arrangement the image in the focal plane consisted of two patterns of fringes which overlapped over a portion of their length. Care was taken to give the Wollaston prism such an orientation that with no field the fringes in the upper pattern were directly in line with those in the lower. It was also arranged that the light forming the upper pattern was polarised by the prism in a plane perpendicular to the lines of force, and that forming the lower pattern polarised in a plane parallel to the field.

On passing a current of small intensity through the electromagnet the main line fringes in the upper pattern became somewhat widened but were not displaced as a whole. On the other hand, each of the main line fringes in the lower pattern became slightly widened and split up into a doublet, one member of the doublet being displaced to the right and the other to the left of the position initially occupied by the fringe.

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* McLennan, 'Roy. Soc. Proc.,' A, vol. 87, p. 273 (1912).

When the field of the electromagnet was gradually increased, the fringes in the upper pattern continued to widen and finally broke up into well-marked triplets. On the other hand, the members of the doublet in the lower pattern continued to separate from each other and to widen, and each finally broke up into a triplet. In this way it was readily shown that the green line broke up into a nonet under moderate fields with the components of the outer triplets polarised parallel to the lines of force and the components of the inner triplet polarised perpendicularly to the lines of force. The method just described also brought out the point that all the members of the nonet were of equal intensity and that they were regularly and equally spaced.

The method of coincidence which was applied consisted in so choosing the magnetic field, that the outer triplet on the left-hand side of one order in the lower pattern exactly overlapped and coincided with the outer triplet on the right-hand side of the next higher order of the same pattern. When fields stronger than those required for coincidence were used, the outer triplets in the lower pattern moved beyond the position of coincidence and produced a confused and indecipherable effect. In the upper pattern the effect of increasing the field was only to spread out and make more marked the members of the triplet.

It is easy to see that when coincidence takes place the change in wave-length, $\Delta\lambda$, of each of the central members of the outer triplets of the nonet relative to that of the undisplaced line is equal to $\frac{1}{2}\Delta\lambda_m$.

For coincidence we have then e/m given by $\frac{e}{m} = \frac{2\pi C\Delta\lambda_m}{H\lambda^2}$. In making a test of this method in which no special care was taken to get a highly accurate result it was found that coincidence was obtained with a field of 6921 gauss. Since for the Lummer plate used $\Delta\lambda_m$ was equal to $0{\cdot}2984 \times 10^{-8}$ cm., it followed that $e/m = 2{\cdot}73 \times 10^7$. From this we have for a normal triplet $e/m = 1{\cdot}82 \times 10^7$, a value which is close to that found in some of the best determinations* with the lines in the mercury arc spectrum.

Summary.

1. The formula $\Delta\lambda = \frac{\alpha_{s_1}{}^2 - \alpha_m{}^2}{\alpha_{m_1}{}^2 - \alpha_m{}^2}\Delta\lambda_m$ has been shown to be the one which

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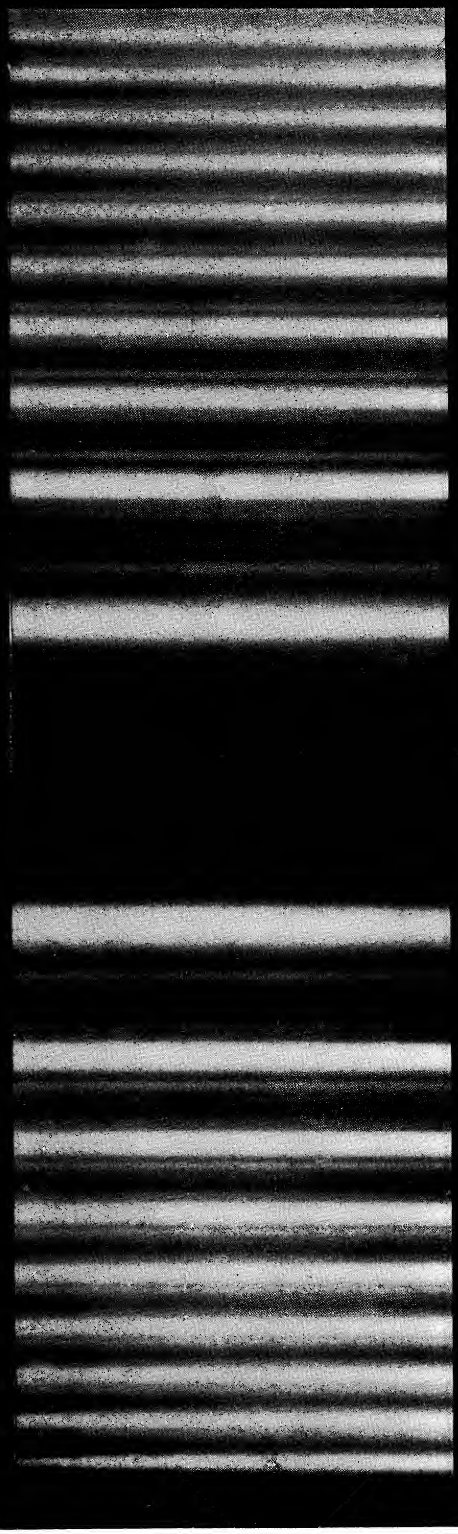
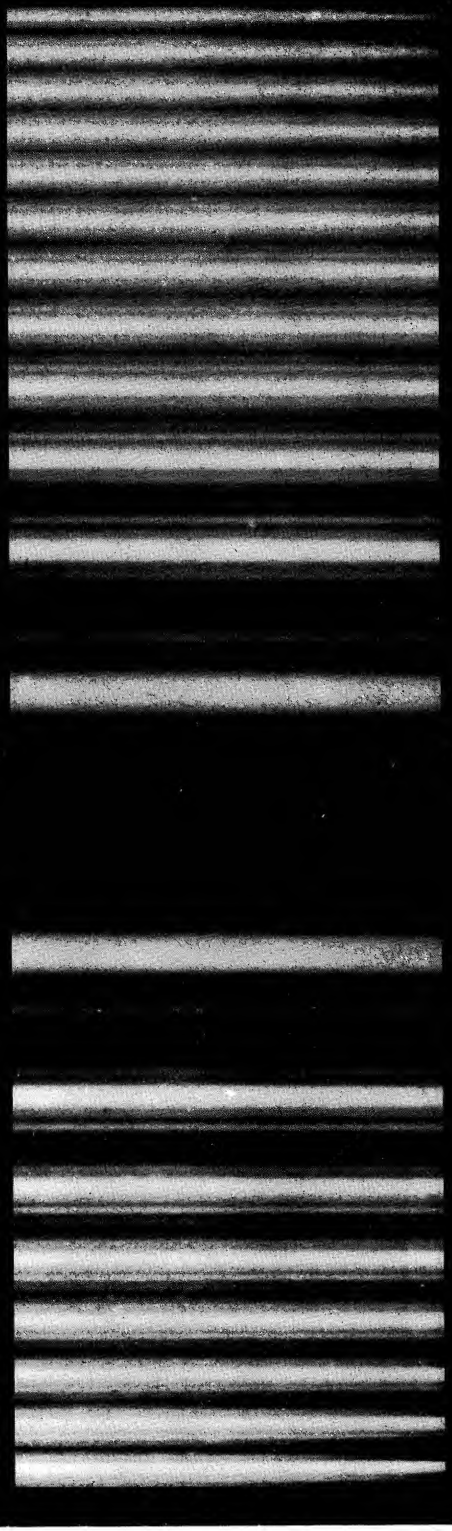
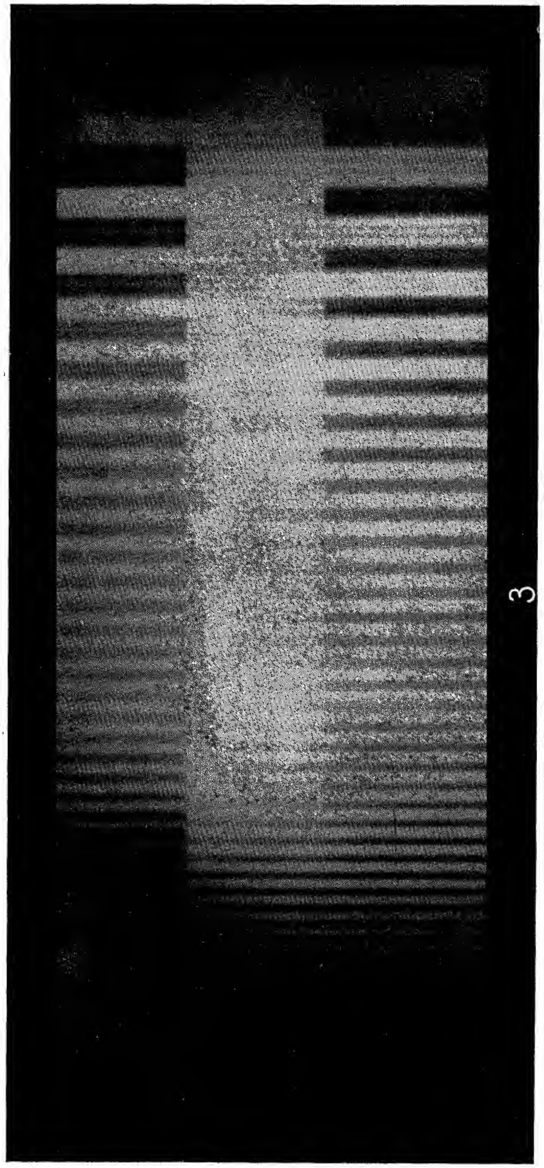
* Nagaoka, 'Nature,' May 5, 1910, p. 377.



1



2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should be used in measuring up the fringe pattern produced by Lummer plates for determining small changes in wave-length.

2. This formula has been applied to the determination of the positions of the satellites of the mercury line $\lambda = 5461$ Å.U. relative to the main component, when the line was resolved by a Lummer plate made of crystal quartz and also by two others made of glass. The results obtained with the quartz plate agree with the values recently found by Nagaoka and Takamine, who used a glass Lummer plate crossed with an échelon grating, while those obtained with the glass Lummer plates agree best with determinations previously made by one of the writers with an échelon grating.

3. Considerations have been presented in favour of the view that the green line consists of a series of lines closing together towards the centre, their position being regulated by some law which in the outer components simplifies approximately to a geometric progression. Evidence is also adduced in support of the view that a somewhat similar law applies approximately to the mercury arc line $\lambda = 3341$ Å.U.

4. A method of coincidence was used to illustrate the magnetic resolution of the mercury green line $\lambda = 5461$ Å.U. and a value of $1{\cdot}82 \times 10^7$ has been obtained for e/m.

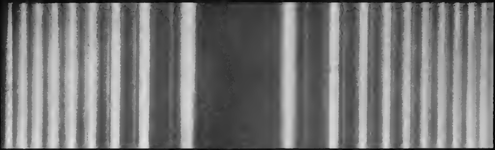
In conclusion the writers wish to convey their thanks to Mr. P. Blackman and Mr. D. A. Keys for assistance in taking the photographs and in checking up the measurements of some of the plates.

DESCRIPTION OF PLATES.

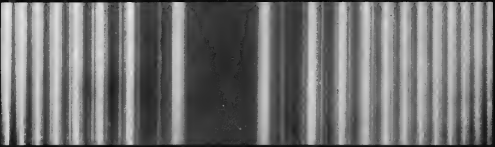
Fig. 1 illustrates the fringe pattern obtained with the quartz Lummer plate. Proceeding outwards from the central portion of the pattern either to the right or to the left, the significance of the fringes between any two orders of main line fringes is as follows: Main line, positive satellite No. 1, positive No. 2, negative No. 4, negative No. 3, positive No. 3, and main line.

In fig. 2 the pattern obtained with one of the glass Lummer plates is reproduced. The order of the fringes in this pattern, proceeding outwards, is as follows:—Main line, negative satellite No. 5, positive No. 1, positive No. 2, negative No. 4, negative No. 3, and main line.

Fig. 3 illustrates the magnetic resolution of the green line $\lambda = 5461$ Å.U. under the conditions described. The line is broken up into a nonet, and the members of the outer triplets of any order are in coincidence with the members of the outer triplets of the next adjacent order.



1



2

Fig. 1 illustrates the fringe pattern obtained with the quartz Lummer plate. Proceeding outwards from the central portion of the pattern either to the right or to the left, the significance of the fringes between any two orders of main line fringes is as follows: Main line, positive satellite No. 1, positive No. 2, negative No. 4, negative No. 3, positive No. 3, and main line.

In fig. 2 the pattern obtained with one of the glass Lummer plates is reproduced. The order of the fringes in this pattern, proceeding outwards, is as follows:—Main line, negative satellite No. 5, positive No. 1, positive No. 2, negative No. 4, negative No. 3, and main line.

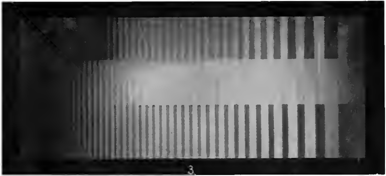


Fig. 3 illustrates the magnetic resolution of the green line $\lambda = 5461$ Å.U. under the conditions described. The line is broken up into a nonet, and the members of the outer triplets of any order are in coincidence with the members of the outer triplets of the next adjacent order.